JOURNAL PUNJAB ACADEMY OF SCIENCES (Peer Reviewed Open Access Journal)
JPAS, 23: 164-174 (January-December, 2023)
Available online at www.jpas.in
ISSN 2229-7014 (P)

MULTI-INDEX TRANSPORTATION PROBLEM: AN OVERVIEW OF ITS VARIANTS, SOLUTION TECHNIQUES AND APPLICATIONS

Bikramjit Singh[1] and Amarinder Singh[2]
[1]Department of Mathematics
Maharaja Ranjit Singh Punjab Technical University, Bathinda (Punjab), India
biks.asr@gmail.com, 2Department of Applied Sciences
Baba Banda Singh Bahadur Engineering College, Fatehgarh Sahib (Punjab), India
amarinder77@gmail.com.

ABSTRACT

Multi-index Transportation Problem (MTP) and its variants have originated to model one of the complex scenarios of transportation problem where cost and/or time of transportation also depend upon type of transported commodity and/or freight vehicle used in transportation along with given sources and destinations. In almost last six and half decades, researchers have extensively studied MTP and its variants from different perspectives, developed many solution approaches and established its applicability by modeling many problems of engineering and management as MTP. However, a comprehensive review of the problem domain is surprisingly absent from the concerned literature, therefore, this paper strives to fill this void by systematically examining 59 published articles filtered through a well-defined search criterion. Methodologically, the examined articles are categorized into various classes based on two, possibly exhaustive but certainly not exclusive, attributes of classification, one is, approach used to study the problem and, another is, contribution of the study. Classification is presented in a two-way tabular form that will serve as a guiding map to young researchers who are intending to study MTP, its variants, and/or applications in engineering and management. This paper also provides an indication for the future work. It concludes that, since its inception, MTP has evolved as one of the very important problem domains in logistics research and metaheuristic approach has contributed most in its study.

Keywords: Multi-index Transportation Problem, Solution Approaches, Applications, Systematic Review, Logistics Research

---

INTRODUCTION

The model of classical transportation problem has been generalized into MTP to formulate a scenario of transportation that intends to minimize the total transportation cost and/or time of transporting certain heterogeneous types of commodities from a fixed number of sources to a definite number of destinations under some specific constraints on available supply and raised demand at each source and destination respectively (Schell 1955, Haley, New methods in mathematical programming—the solid transportation problem 1962, Haley, The multi-index problem 1963). In last few decades, continuously growing use of technology and transportation network in logistics services have expanded the horizon of its applications to disaster management (Sarma, et al. 2020, Sengupta, et al. 2021), production planning (Afraimovich and Prilutskii 2010), data transportation (Ramane and Francis 2011) etc., and consequently, motivated researchers to formulate new variants of existing transportation models, propose more efficient solution techniques and find their applications to existing and upcoming problems in management and engineering (Khurana, Adlakha and Lev, Multi-index constrained transportation problem with bounds on availabilities, requirements and commodities 2018, Lev, Oksana and Viacheslav 2017, Ojha, et al. 2010, Das, Roy and Weber 2020, Wang, Tizik and Tsurkov, Decomposition Algorithm for the Linear Three-Index Transportation Problem 2019, Singh and Singh, Hybrid particle swarm optimization for pure integer linear solid transportation problem 2023).

Although, the basic model of MTP was introduced in 1955 (Schell 1955), the necessary conditions for the existence of its feasible solution and first analytical method, namely extended Modified Distribution (MODI) method, to solve it were studied, much later, between years 1962 and 1986 (Haley, The multi-index problem 1963, Haley, The existence of a solution to the multi-index problem 1965, Moravek and Vlach 1967, Smith, A procedure for determining necessary and sufficient conditions for the existence of a solution to the multi-index problem 1974, Smith, Further necessary conditions for the existence of a solution to the multi-index problem 1973, Vlach 1986, Haley, New methods in mathematical programming—the solid transportation problem 1962). In last two decades, the growing size and applications of logistics service industry have initiated many researchers to develop more computationally efficient algorithmic solution techniques, by using various theoretical approaches, to determine exact or near exact solutions to large size NP-hard MTPs in reasonable time.

In the concerned literature, five main theoretical approaches used to study MTP can be recognised. First approach is the analytical or classical provable algorithmic approach that considers MTP as Linear Programming Problem (LPP) which, later, are extended to some non-linear variants of MTP as well. Methods developed with this approach include Simplex/Revised Simplex Method, Modified Distribution Method (MODI method), Goal Programming technique to solve multi-objective MTP (Haley, New methods in mathematical programming—the solid transportation problem 1962, Khurana and Adlakha, On multi-index fixed charge bi-criterion transportation problem 2015), moreover, conditions of optimality and necessary conditions for the existence of feasible/admissible solution to MTP (Haley, The multi-index problem 1963, Khurana and Adlakha, On multi-index fixed charge bi-criterion transportation problem 2015, Vlach 1986) are also studied with the same approach. Second main approach is metaheuristic approach to search near-optimal solution to NP-hard MTP in computationally reasonable time in case polynomial time algorithms are not available due to large size and/or complex objective and/or constraint functions of the problem. It includes Evolutionary Algorithms (genetic algorithms, particle swarm optimization, etc.), Lagrangian Relaxation, Electromagnetism-like Algorithm, etc (Akbari, Molla-Alizadeh-Zavardehi and Niroomand 2020, Sanei, Mahmoodirad and Niroomand, et al. 2017). Third is the decompositional approach that decomposes main problem into problems of lower index (called sub-problems) which are further solved either sequentially or in parallel with some efficient method (analytical or heuristic), hence, sequentially converge to optimum solution of main problem (Wang, Tizik and Tsurkov, Decomposition Algorithm for the Linear Three-Index Transportation Problem 2019, L. G. Afraimovich 2014). It includes Branch and Bound method, Dantzig-Wolfe decomposition algorithm, Benders decomposition technique, etc. It is important to note that commercial optimization solvers like IBM ILOG CPLEX solves MTP and some of its variants by modelling them as Mixed Integer Programming Problem (MIPP) and then using decomposition approach (Kalvelagen 2002). Fourth less used approach is Neural network approach that is applied on single as well as multi-objective MTPs to solve them (Ida, Gen and Li 1996, Li, et al. 1997). Fifth and last approach transforms some variants of MTP to the standard model of MTP that can be solved by applying any one of the solution techniques described in other four approaches (Khurana, Adlakha and Lev, Multi-index constrained transportation problem with bounds on availabilities, requirements and commodities 2018, Junginger 1993).

In addition to above mentioned five approaches, MTP has been studied from the perspective of theoretical computer science and polyhedral combinatorics. This approach considers each solution of MTP as three-dimensional transportation polytope, that is equivalent to a three-way statistical table or contingency table, and hence is inclined to study various properties like existence, counting, and computational complexity, etc. of these polytopes or tables which satisfy some specified constraints on the sums of numbers (called marginals) in their cells (De Loera and Onn 2004, Korsnikov and Burkard 1989, Benson-Putnins 2014, Barvinok 2017).

Till the date, only two significant review articles have covered MTP. One of these articles has presented comprehensive review on classical transportation problems, its variants, and all its extensions including MTP and a few of its variants (Kacher and Singh 2021) and another has covered extensions of MTP in fuzzy and stochastic environment only (Singh, Renu and Sarode, A review on fuzzy and stochastic extensions of the Multi Index transportation problem 2016). Main limitation of reference (Kacher and Singh 2021) is that it tried to cover MTP as an extension of classical transportation problem which itself has very wide range of variants, extensions, solution methods, and applications so it neither succeeded to cover all important variants, approaches and applications of MTP nor in presenting classification of research articles cited for MTP. On the other hand, reference (Singh, Renu and Sarode, A review on fuzzy and stochastic extensions of the Multi Index transportation problem 2016) has focused on MTP in fuzzy environment only and hence has not covered MTP in classical or crisp environment.

This paper presents an overview of three major contributions – variants, solution techniques and applications of MTP, by thoroughly examining 59 articles published in various research journals, conference proceedings, and books in last six and half decades. All examined articles are filtered through a well-defined search criterion, categorically sorted based on five important approaches in studying MTP and presented in a two-way table. Some important articles are highlighted for special attention of the readers and an indication for future work is given. Paper is structured in total six sections including introduction. Section 2 covers methodology used to search, sort, and present examined articles, section 3 presents brief background of the topic, section 4 presents two-way table of classification, section 5 discusses major observations of this brief survey and at last conclusion is given.

Methodology

Criterion to search and filter articles

To search and select research articles related to the field, an online research assistant, named, “Elicit” (Ought 2023) is used that searches relevant articles even if their keywords do not match the search string. “Elicit” allows to search papers with specific keywords included and excluded in their abstracts. Search criteria used are detailed in **Error! Reference source not found.**to reproduce the same results as the author has obtained.

Table 1: Search Criteria using “ELICIT”

Main Search String	Keywords included in Abstract	Keywords excluded from Abstract
Evolutionary algorithms for "multi-index transportation problem"	Three-index transportation problem	Fuzzy, polytopes, stochastic, uncertain, interval number
	Multi-index transportation problem	
	Solid transportation problem	
Study of three-index transportation polytopes	Transportation polytope, three-index	Fuzzy, stochastic, uncertain
	Transportation polytope, multi-index	

Criterion for classification of selected articles

Classification of selected articles is based on two attributes namely theoretical approach used in it, and its major contributions. After examining selected articles, it has been identified that researchers have made three major contributions to the problem domain of MTP by using five main theoretical approaches.

Necessary background

Definition of MTP

Multi-index Transportation Problem is an optimization problem of minimizing the total cost for transporting heterogeneous commodities from a fixed number of sources to certain destinations under the constraints that supplies of each type of commodity at each source and demands of each type of goods at every destination are preset.

Mathematical Model of MTP

The parameters and decision variables used to formulate the standard model MTP are as under:

Parameters -

- $m, n,$ and p are the number of sources, destinations, and types of commodities respectively,
- $i,\ j,$ and k are the indices for source, destination, and type of commodity respectively where $1 \leq i \leq m$, $1 \leq j \leq n$, and $1 \leq k \leq p$,
- c_{ijk} is per unit cost of transporting k^{th} type of commodity from i^{th} source to j^{th} destination,
- A_{ki} is the total amount of k^{th} type of commodity available at i^{th} source,
- B_{jk} is the total amount of k^{th} type of commodity transported to j^{th} destination,
- E_{ij} is the amount of all types of commodities transported from i^{th} source to j^{th} destination.

Decision variable –

- x_{ijk} is the amount of k^{th} type of commodity transported from i^{th} source to j^{th} destination,

With above defined notations, the mathematical model of MTP is:

$$Minimize \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{p} c_{ijk}\, x_{ijk} \quad (Objective\ function)$$

Subject to the constraints:

$$\sum_{j=1}^{n} x_{ijk} = A_{ki} \quad \forall\, k, i \quad (Supply\ Constraints)$$

$$\sum_{i=1}^{m} x_{ijk} = B_{jk} \quad \forall\, j, k \quad (Demand\ Constraints)$$

$$\sum_{k=1}^{p} x_{ijk} = E_{ij} \quad \forall\, i, j \quad (Commodity\ Constraints)$$

$$x_{ijk} \geq 0 \quad \forall\, i, j, k \quad (Feasibility\ Constraints)$$

Where $[A_{ki}]_{p\times m}$, $[B_{jk}]_{n\times p}$, and $[E_{ij}]_{m\times n}$ are rectangular matrices of non-negative numbers.

Variants of MTP

In literature, MTP has been formulated into various other mathematical forms to address spectrum of practical situations shown up while modeling it. These other models of MTP are called its variants and a few of these are fixed charge MTP, fixed charge bi-criteria MTP, step fixed charge solid transportation problem, time minimization MTP, non-linear MTP, multi-index constrained transportation problem with bounds on availabilities, requirements and commodities, solid transportation-p-facility location problem, etc. Mathematically, these models vary from each other either in the type of objective function, i.e., linear or non-linear, or in number of objective functions, i.e., single or multi-objective, or in types of constraint functions, i.e., single sum or double sum or bounds on constraints, etc.

CLASSIFICATION OF REVIEWED ARTICLES

All the articles selected for analysis have been divided into different classes using two attributes which are detailed in next two subsections.

Contributions of reviewed articles

Each of the selected articles has been examined to recognize its contribution in the following three major categories, and fourth category is created to cover other miscellaneous contributions:

C1-Proposed either the standard model or a new variant of MTP,

C2-Proposed a method or technique to solve standard MTP or any of its variant,

C3-Proposes an application of standard MTP, or any of its variant; modeling a problem of engineering and management as MTP,

C4-Miscellaneous like sensitivity analysis, necessary conditions for feasible solution of MTP, etc.

Approaches used in studying MTP

Further, it is recognized that selected articles have made four different types of contributions by using five main theoretical approaches that are as follows:

A1- Analytical or classical provable algorithmic approach,

A2- Metaheuristic approach,

A3- Decompositional approach,

A4- Neural network approach,

A5- Approach to transform a particular variant of MTP into standard MTP or network flow problem that can be solved by using any existing solution technique.

Tabular presentation

Two-way classification is presented in Table2 where five approaches are aligned in five horizontal rows and four categories of contributions are assigned to four vertical columns of it.

DISCUSSION

Classification Table2 reveals that selected articles have contributed 10 new variants, 11 different solution techniques, and 1 application using approach A1- whereas 7 articles are dedicated study necessary conditions for feasible solution to MTP. At the same time, unfolds that 9 distinct variants, 16 solution techniques, 5 applications, and miscellaneous contributions have been studied with approach A2-. Similarly, the classification table reveals major contributions of selected articles using approaches A3- to A5-.

CONCLUSION

This paper dedicatedly, not exhaustively, searched and skimmed maximum possible research articles using well-defined search strings and keywords with the help of an AI based online research assistant and Google Scholar. All the selected articles are then examined manually and arranged in a two-way classification table using two different attributes. The amount and variety of research publications dedicated to study of MTP by using various theoretical approaches has evolved it as one of the very important problem domains in logistics research.

The classification table reveals that most of the contributions in terms of new variants, solution techniques and applications of MTP are made by using two approaches of study, namely analytical and metaheuristic whereas maximum techniques to solve MTP and its variants, and to study its applications are developed using metaheuristic approach. This points that metaheuristic approach has emerged as one of the very significant approaches considered in this paper. However, the limited scope and study of this paper do not allow to undermine the contributions of other approaches. It is important to note that decomposition approach and the approach of transforming variants of MTP into standard MTP or network flow problems always need either analytical or metaheuristic or neural network techniques to find the solution of MTP in hand, hence, in future these approaches may be considered in simultaneity to the study of metaheuristic techniques.

In future, this precise review can be extended to most comprehensively systematic review of MTP where very precise contribution of each article can be presented.

TABLE 2. TWO-WAY CLASSIFICATION

Approach	Contribution			
	C1-	***C2-***	***C3-***	***C4-***
A1-	(Haley, New methods in mathematical programming—the solid transportation problem 1962), (Haley, The multi-index problem 1963), (Bhatia, Indefinite quadratic solid transportation problem 1981), (Basu and Acharya 2002), (Bandopdhyaya 1985), (Acharya 2016), (Singh, Chauhan and Kuldeep, Time Minimizing Multi-Index Bulk Transportation 2018), (Bhatia, Swarup and Puri, Time minimizing solid transportation problem 1976), (Chauhan and Khanna 2021), (Tanwar and Chauhan 2020) **(10 papers)**	(Haley, New methods in mathematical programming—the solid transportation problem 1962), (Haley, The multi-index problem 1963), (Khurana and Adlakha, On multi-index fixed charge bi-criterion transportation problem 2015), (Bhatia, Indefinite quadratic solid transportation problem 1981), (Basu and Acharya 2002), (Acharya 2016), (Singh, Chauhan and Kuldeep, Time Minimizing Multi-Index Bulk Transportation 2018), (Bhatia, Swarup and Puri, Time minimizing solid transportation problem 1976), (Chauhan and Khanna 2021), (Basu, Pal and Kundu 1994), (Pandian and Anuradha, A new approach for solving solid transportation problems 2010) **(11 papers)**	(Sengupta, et al. 2021) **(1 paper)**	References (Haley, The multi-index problem 1963), (Haley, The existence of a solution to the multi-index problem 1965), (Moravek and Vlach 1967), (Smith, A procedure for determining necessary and sufficient conditions for the existence of a solution to the multi-index problem 1974), (Smith, Further necessary conditions for the existence of a solution to the multi-index problem 1973), (Vlach 1986), (Matveenko 1986) studies necessary conditions for the existence of feasible solution to MTP; Reference (Pandian and Kavitha, Sensitivity analysis in solid transportation problems 2012) proposed a technique to find two types of costs sensitivity range **(7 papers)**
A2	(Sarma, et al. 2020), (Ojha, et al. 2010), (Das, Roy and Weber 2020), (Khurana and Adlakha, On multi-index fixed charge bi-criterion transportation problem 2015), (Akbari, Molla-Alizadeh-Zavardehi and Niroomand 2020), (Sanei, Mahmoodirad and Niroomand, et al. 2017), (Oyewole and	(Sarma, et al. 2020), (Ramane and Francis 2011), (Ojha, et al. 2010), (Das, Roy and Weber 2020), (Singh and Singh, Hybrid particle swarm optimization for pure integer linear solid transportation problem 2023), (Akbari, Molla-Alizadeh-Zavardehi and Niroomand 2020), (Sanei, Mahmoodirad and Niroomand, et al. 2017), (Oyewole and Adetunji 2021), (Paşa 2022), (Sanei, Mahmoodirad and MOLLA, An electromagnetism-like algorithm	(Sarma, et al. 2020), (Ramane and Francis 2011), (Das, Roy and Weber 2020), (Chernov, et al. 2021), (Cao 2022)	Reference (Singh and Singh, Hybrid particle swarm optimization for pure integer linear solid transportation problem 2023) studies the sensitivity of various set of parameter settings in proposed hybrid PSO using well-defined methodology **(1 paper)**

	Adetunji 2021), (Paşa 2022), (Cao 2022) **(9 papers)**	for fixed charge solid transportation problem 2013), (Gen, Ida and Li 1994), (Singh and Singh, Hybrid Particle Swarm Optimization Algorithm to Solve Multi-Index Fixed Charge Transportation Problem 2021), (Skitsko and Voinikov 2020), (Singh, Singh and Singh 2021), (Dimov 2016), (Chernov, et al. 2021) **(16 papers)**	**(5 papers)**	
A3-	(Lev, Oksana and Viacheslav 2017), (L. G. Afraimovich 2014) **(2 papers)**	(Lev, Oksana and Viacheslav 2017), (Wang, Tizik and Tsurkov, Decomposition Algorithm for the Linear Three-Index Transportation Problem 2019), (L. G. Afraimovich 2014), (Wang, Esenkov, et al. 2018), (Abd Elazeem, et al. 2021) **(5 papers)**	---	---
A4-	---	(Ida, Gen and Li 1996), (Li, et al. 1997) **(2 papers)**	---	---
A5-	(Khurana, Adlakha and Lev, Multi-index constrained transportation problem with bounds on availabilities, requirements and commodities 2018), (Bandopdhyaya 1985) **(2 papers)**	---	(Afraimovich and Prilutskii 2010), (Afraimovich and Prilutskii 2006), (Tan and Wu 2012) **(3 papers)**	---

REFERENCES

Abd Elazeem, Abd Elazeem M., Abd Allah A. Mousa, Mohammed A. El-Shorbagy, Sayed K. Elagan, and

Yousria Abo-Elnaga. 2021. "Detecting All Non-Dominated Points for Multi-Objective Multi-Index Transportation Problems." *Sustainability* (MDPI) 13: 1372.

Acharya, Debiprasad. 2016. "Generalized solid capacitated transportation problem." *South Asian J Math* 6: 24–30.

Afraimovich, L. G., and M. Kh Prilutskii. 2010. "Multiindex optimal production planning problems." *Automation and remote control* 71: 2145.

Afraimovich, Lev G. 2014. "A heuristic method for solving integer-valued decompositional multiindex problems." *Automation and Remote Control* (Springer) 75: 1357–1368.

Afraimovich, Lev Grigor'evich, and M. Kh Prilutskii. 2006. "Multiindex resource distributions for hierarchical systems." *Automation and remote control* (Springer) 67: 1007–1016.

Akbari, Mojtaba, Saber Molla-Alizadeh-Zavardehi, and Sadegh Niroomand. 2020. "Meta-heuristic approaches for fixed-charge solid transportation problem in two-stage supply chain network." *Operational Research* (Springer) 20: 447–471.

Bandopdhyaya, Lakshmisree. 1985. "The capacitated solid transportion problem (planar constraints) with upper and lower bounds on rim conditions." *Journal of the Korean Operations Research and Management Science Society* (The Korean Operations Research and Management Science Society) 10: 45–53.

Barvinok, Alexander. 2017. "Counting integer points in higher-dimensional polytopes." In *Convexity and concentration*, 585–612. Springer.

Basu, M., B. B. Pal, and A. Kundu. 1994. "An algorithm for finding the optimum solution of solid fixed-charge transportation problem." *Optimization* (Taylor & Francis) 31: 283–291.

Basu, Manjusri, and Debi Prasad Acharya. 2002. "On quadratic fractional generalized solid bi-criterion transportation problem." *Journal of Applied Mathematics and Computing* (Springer) 10: 131–143.

Benson-Putnins, David. 2014. "Counting integer points in multi-index transportation polytopes." *arXiv preprint arXiv:1402.4715.*

Bhatia, H. L. 1981. "Indefinite quadratic solid transportation problem." *Journal of Information and Optimization Sciences* (Taylor & Francis) 2: 297–303.

Bhatia, H. L., Kanti Swarup, and M. C. Puri. 1976. "Time minimizing solid transportation problem." *Statistics: A Journal of Theoretical and Applied Statistics* (Taylor & Francis) 7: 395–403.

Cao, Junfang. 2022. "Mathematical Model and Algorithm of Multi-Index Transportation Problem in the Background of Artificial Intelligence." *Journal of Advanced Transportation* (Hindawi) 2022.

Chauhan, Sudhir Kumar, and Pallavi Khanna. 2021. "Efficient Solution for Bi-Criteria Multi-Index Bulk Transportation Problem." *2021 9th International Conference on Reliability, Infocom Technologies and Optimization (Trends and Future Directions)(ICRITO).* 1–4.

Chernov, Serhii, Serhii Titov, Liudmyla Chernova, Varvara Piterska, Liubava Chernova, and Nataliia Kunanets. 2021. "Three-index optimization transportation model." *2021 IEEE 16th International Conference on Computer Sciences and Information Technologies (CSIT).* 315–318.

Das, Soumen Kumar, Sankar Kumar Roy, and Gerhard Wilhelm Weber. 2020. "Heuristic approaches for solid transportation-p-facility location problem." *Central European Journal of Operations Research* (Springer) 28: 939–961.

De Loera, Jesus, and Shmuel Onn. 2004. "The complexity of three-way statistical tables." *SIAM Journal on Computing* (SIAM) 33: 819–836.

Dimov, Yuri Sergeevich, and Nikita Dmitrievich Lukyanov. 2016. "Application of a genetic algorithm for solving a triplanar transport problem." *Bulletin of the Irkutsk State Technical University* 114 (7): 73-79.

Gen, Mitsua, Kenichi Ida, and Yinzhen Li. 1994. "Solving bicriteria solid transportation problem by genetic algorithm." *Proceedings of IEEE international conference on systems, man and cybernetics.* 1200–1207.

Haley, K. B. 1962. "New methods in mathematical programming—the solid transportation problem." *Operations research* (INFORMS) 10: 448–463.

Haley, K. B. 1965. "The existence of a solution to the multi-index problem." *Journal of the Operational Research Society* (Taylor & Francis) 16: 471–474.

Haley, K. B. 1963. "The multi-index problem." *Operations research* (INFORMS) 11: 368–379.

Ida, Kenichi, Mitsuo Gen, and Yinzhen Li. 1996. "Neural networks for solving multicriteria solid transportation problem." *Computers & industrial engineering* (Elsevier) 31: 873–877.

Junginger, Werner. 1993. "On representatives of multi-index transportation problems." *European journal of operational research* (Elsevier) 66 (3): 353--371. doi:https://doi.org/10.1016/0377-2217(93)90223-A.

Kacher, Yadvendra, and Pitam Singh. 2021. "A comprehensive literature review on transportation problems." *International Journal of Applied and Computational Mathematics* (Springer) 7: 1–49.

Kalvelagen, Erwin. 2002. "Benders decomposition with GAMS." *Amsterdam Optimization Modeling Group: Washington, DC, USA.* http://www.amsterdamoptimization.com/pdf/benders.pdf.

Khurana, Archana, and Veena Adlakha. 2015. "On multi-index fixed charge bi-criterion transportation problem." *Opsearch* (Springer) 52: 733–745.

Khurana, Archana, Veena Adlakha, and Benjamin Lev. 2018. "Multi-index constrained transportation problem with bounds on availabilities, requirements and commodities." *Operations Research Perspectives* (Elsevier) 5: 319–333.

Korsnikov, A. D., and E. E. Burkard. 1989. "On the dimension of polytopes of planar three-index transportation problems." *Optimization* (Taylor & Francis) 20: 107–116.

Lev, Raskin, Sira Oksana, and Karpenko Viacheslav. 2017. "Calculation of throughputs of intermediate centers in three-index transportation problems." *Восточно-Европейский журнал передовых технологий* (Частное предприятие \guillemotleftТехнологический центр\guillemotright) 3: 31–37.

Li, Yinzhen, Kenichi Ida, Mitsuo Gen, and Reiko Kobuchi. 1997. "Neural network approach for multicriteria solid transportation problem." *Computers & industrial engineering* (Elsevier) 33: 465–468.

Matveenko, Vladimir Dmitrievich. 1986. "A combinatorial approach to the problem of solvability of the three-index transportation problem." *Mathematical Notes of the Academy of Sciences of the USSR* (Springer) 40: 638–643.

Moravek, J., and M. Vlach. 1967. "On the necessary conditions for the existence of the solution of the multi-index transportation problem." *Operations Research* (INFORMS) 15: 542–545.

Ojha, Anupam, Barun Das, S. Mondal, and Manoranjan Maiti. 2010. "A solid transportation problem for an item with fixed charge, vechicle cost and price discounted varying charge using genetic algorithm." *Applied Soft Computing* (Elsevier) 10: 100–110.

Ought. 2023. *Elicit: The AI Research Assistant.* https://elicit.org, Feb 22.

Oyewole, Gbeminiyi John, and Olufemi Adetunji. 2021. "Solving the facility location and fixed charge solid transportation problem." *Journal of Industrial and Management Optimization* (Journal of Industrial and Management Optimization) 17: 1557–1575.

Pandian, P., and D. Anuradha. 2010. "A new approach for solving solid transportation problems." *Applied mathematical sciences* 4: 3603–3610.

Pandian, P., and K. Kavitha. 2012. "Sensitivity analysis in solid transportation problems." *Applied Mathematical Sciences* 6: 6787–6796.

Paşa, Tatiana. 2022. "Solving the non-linear multi-index transportation problems with genetic algorithms." *Computer Science Journal of Moldova* 88: 77–92.

Ramane, Muralikrishnan, and F. Sagayaraj Francis. 2011. "A Multi-Commodity Grid Logistics Model for Data Transportation in Federated Datagrid." *International Journal of Computer Applications* (International Journal of Computer Applications, 244 5 th Avenue,# 1526, New …) 975: 8887.

Sanei, Masoud, Ali Mahmoodirad, and A.L.I.Z.A.D.E.H. Z.A.V.A.R.D.E.H.I. S. MOLLA. 2013. "An electromagnetism-like algorithm for fixed charge solid transportation problem." (INTERNATIONAL JOURNAL OF MATHEMATICAL MODELLING & COMPUTATION).

Sanei, Masoud, Ali Mahmoodirad, Sadegh Niroomand, Ali Jamalian, and Shahin Gelareh. 2017. "Step fixed-charge solid transportation problem: a Lagrangian relaxation heuristic approach." *Computational and Applied Mathematics* (Springer) 36: 1217–1237.

Sarma, Deepshikha, Amrit Das, Pankaj Dutta, and Uttam Kumar Bera. 2020. "A cost minimization resource allocation model for disaster relief operations with an information crowdsourcing-based MCDM approach." *IEEE transactions on engineering management* (IEEE) 69: 2454–2474.

Schell, E. D. 1955. "Distribution of a product by several properties." *Proceedings of 2nd Symposium in Linear Programming.* Washington D C: DCS/comptroller, HQ US Air Force.

Sengupta, Dipanjana, Amrit Das, Uttam Kumar Bera, and Anirban Dutta. 2021. "A Humanitarian Green Supply Chain Management Considering Minimum Cost and Time." *International Journal of Business Analytics (IJBAN)* (IGI Global) 8: 63–82.

Singh, Bikramjit, and Amarinder Singh. 2021. "Hybrid Particle Swarm Optimization Algorithm to Solve Multi-Index Fixed Charge Transportation Problem." *2021 4th International Conference on Computing and Communications Technologies (ICCCT).* 584–589.

Singh, Bikramjit, and Amarinder Singh. 2023. "Hybrid particle swarm optimization for pure integer linear solid transportation problem." *Mathematics and Computers in Simulation* (Elsevier) 207: 243–266.

Singh, Gurwinder, Amarinder Singh, and Bikramjit Singh. 2021. "Incorporation of additional modules within particle swarm optimization for feasibility conditions of multi-index transportation problem." *Hybrid Intelligent Systems: 20th International Conference on Hybrid Intelligent Systems (HIS 2020), December 14-16, 2020.* 780–792.

Singh, Sungeeta, Sudhir K. Chauhan, and Kuldeep. 2018. "Time Minimizing Multi-Index Bulk Transportation." *Journal of Mathematics and Informatics* 13: 13-19.

Singh, Sungeeta, T. U. L. I. Renu, and Deepali Sarode. 2016. "A review on fuzzy and stochastic extensions of the Multi Index transportation problem." *Yugoslav Journal of Operations Research* 27: 3–29.

Skitsko, Volodymyr, and Mykola Voinikov. 2020. "Solving four-index transportation problem with the use of a genetic algorithm." *LogForum* 16.

Smith, Graham. 1974. "A procedure for determining necessary and sufficient conditions for the existence of a solution to the multi-index problem." *Aplikace matematiky* (Institute of Mathematics, Academy of Sciences of the Czech Republic) 19: 177–183.

Smith, Graham. 1973. "Further necessary conditions for the existence of a solution to the multi-index problem." *Operations Research* (INFORMS) 21: 380–386.

Tan, Matthias H. Y., and C.F. Jeff Wu. 2012. "Generalized selective assembly." *IIE Transactions* (Taylor & Francis) 44: 27–42.

Tanwar, Kuldeep, and Sudhir Kumar Chauhan. 2020. "Time-Cost Solution Pairs in Multi-index Bulk Transportation Problem." *Data Science and Analytics: 5th International Conference on Recent Developments in Science, Engineering and*

Technology, REDSET 2019, Gurugram, India, November 15–16, 2019, Revised Selected Papers, Part II 5. 154–163.

Vlach, Milan. 1986. "Conditions for the existence of solutions of the three-dimensional planar transportation problem." *Discrete Applied Mathematics* (North-Holland) 13: 61–78.

Wang, L. P., A. P. Tizik, and V. I. Tsurkov. 2019. "Decomposition Algorithm for the Linear Three-Index Transportation Problem." *Journal of Computer and Systems Sciences International* (Springer) 58: 883–888.

Wang, L. P., A. S. Esenkov, A. P. Tizik, and E. V. Torchinskaya. 2018. "Decomposition Method for Solving Three-Index Transportation Problems." *Journal of Computer and Systems Sciences International* (Springer) 57: 759–765.